Lecture 2
Circuits and Electronics
Chapter 3 and Notes

- Intro to basic circuits principles
- Passive, active circuits; circuit laws
- Filters
- Op amps (ideal, nonideal amplifiers)
- Active circuits with op amps
- Applications and problems

ELECTRONIC COMPONENTS

There are two classes of components: <u>Passive and Active</u>

- **PASSIVE**: Resistors, capacitors, transformer/inductor etc. are known as passive devices. Their properties do not depend actively on currents/voltages applied. Their properties are usually linear.

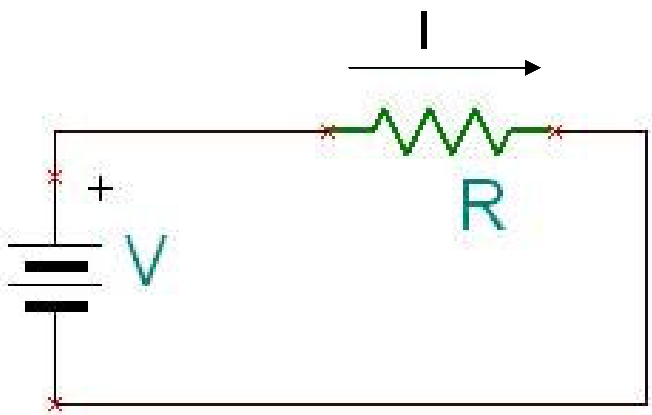
- **ACTIVE**: Diodes, Transistors (various types; analog/digital such as bipolar, field effect used in analog circuit and MOS/CMOS used in digital circuits), LED/ photodetector, op amp, digital integrated circuits. Their properties are usually nonlinear

Another way to classify: Analog vs. Digital

- **ANALOG**: The component response is continuously and proportionally dependent on the input (may be linear or nonlinear). Good example: amplifier

- **DIGITAL**: The component response is essentially 1 or 0 (ON or OFF). These are usually used in digital circuits, computer memory and processors. Good example: Microprocessor

RESISTORS



Ohm's Law

$$V = IR$$

V: volts and I: amperes

Resistor color code: BBROYGBVGW (Black, Brown, Red, Orange, Yellow, Green, Blue, Violet, Gray, White: 0, 1, 2…, 9)

Resistors have tolerance: $\pm$5 or 10% (1% for extra cost)

Power level: ¼ W to many W (W: watts): $P = V * I = I^2 R = V^2/R$

Energy (Joules): $(V^2/r)\ T$

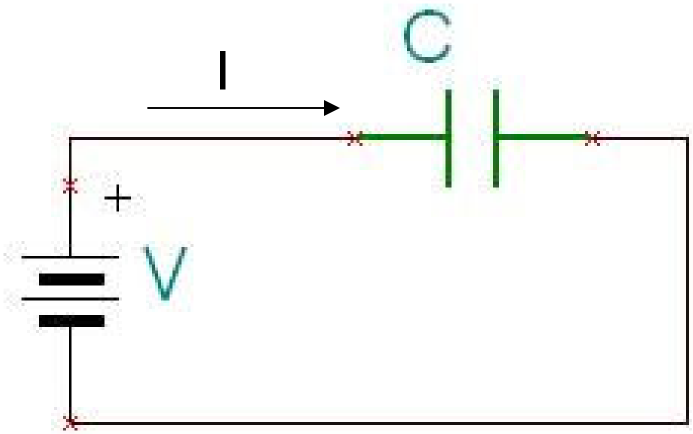
Applications? Light bulb, electric iron, …

CAPACITORS – DC

Stores charge: Q (Coulombs)

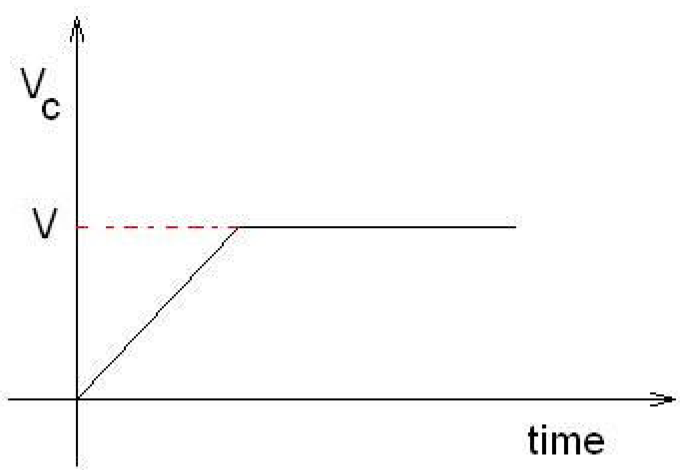
Flow of charge is Current: I (Amperes)

$I = \Delta Q / \Delta T$



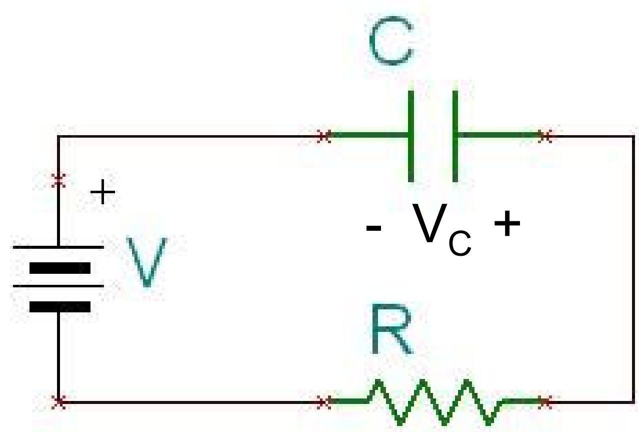
$$I = C\frac{dV_C}{dt}$$

$$V_C = \frac{1}{C}\int i\,dt$$

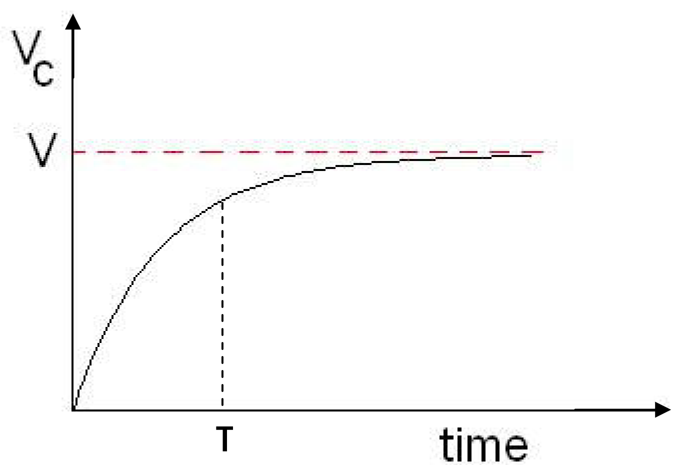


The capacitor charges linearly till the voltage across it reaches the applied voltage after which the driving force is lost and the capacitor ‘blocks’ DC.

RC CIRCUIT – DC

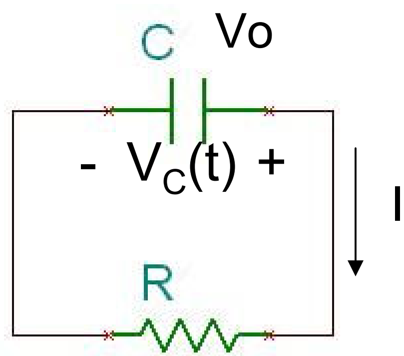


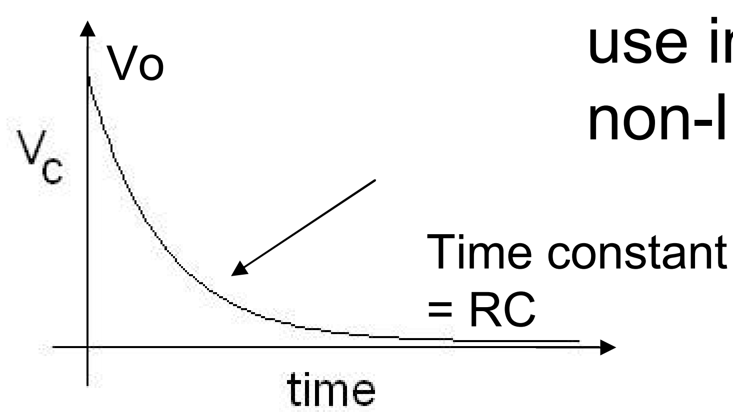
$$V_C(t) = V(1 - e^{-t/RC})$$



This is similar but the capacitor charges non-linearly till the voltage across it reaches the applied voltage after which the driving force is lost. Time constant τ=1/RC is the time in which the capacitor is charged to 67%

RC CIRCUIT – DC



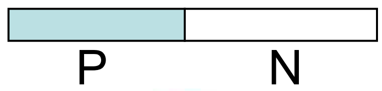


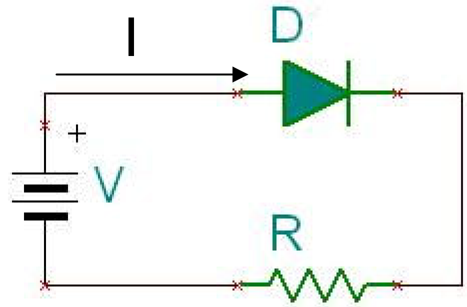
After a capacitor has charged to V_0, it discharges if there is a resistance in the external circuit (otherwise it retains the charge : use in DRAMs). The discharge is non-linear $V_C(t) = V_0 e^{-t/RC}$

We will return to Capacitors in the section 'Impedance' to consider their frequency response.

DIODES

P, N is the “doping” of silicon to carry P (+) or N (-) charge)

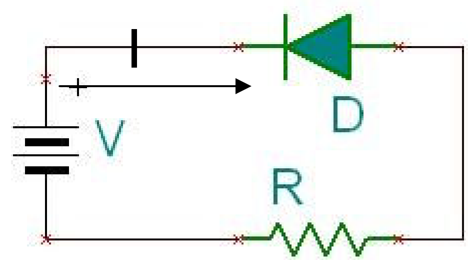




Forward bias, conducting

If V > V_{ON} of diode, $$I = \frac{V - V_{ON}}{R}$$

Von ~ 0.6 V

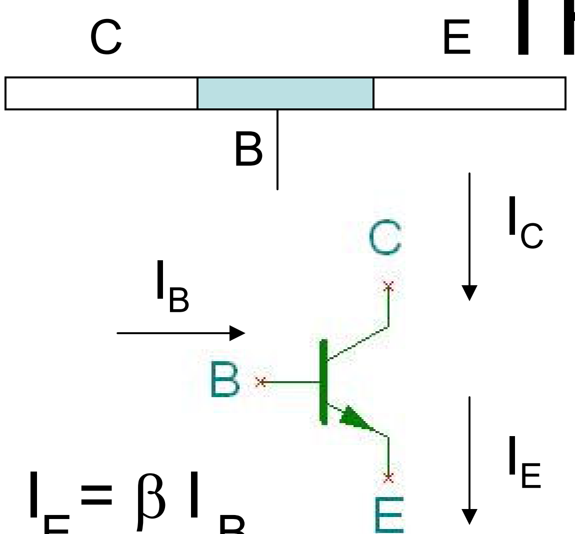


Reverse bias, non conducting

$$I = 0$$

Diodes are silicon based semiconductor devices with P and N junctions. They carry current through electrons or holes (+ charges) in *one direction*.

BIPOLAR JUNCTION TRANSISTORS



$I_E = \beta\, I_B$

Base, Emitter, Collector

$$I_E = I_B + I_C$$

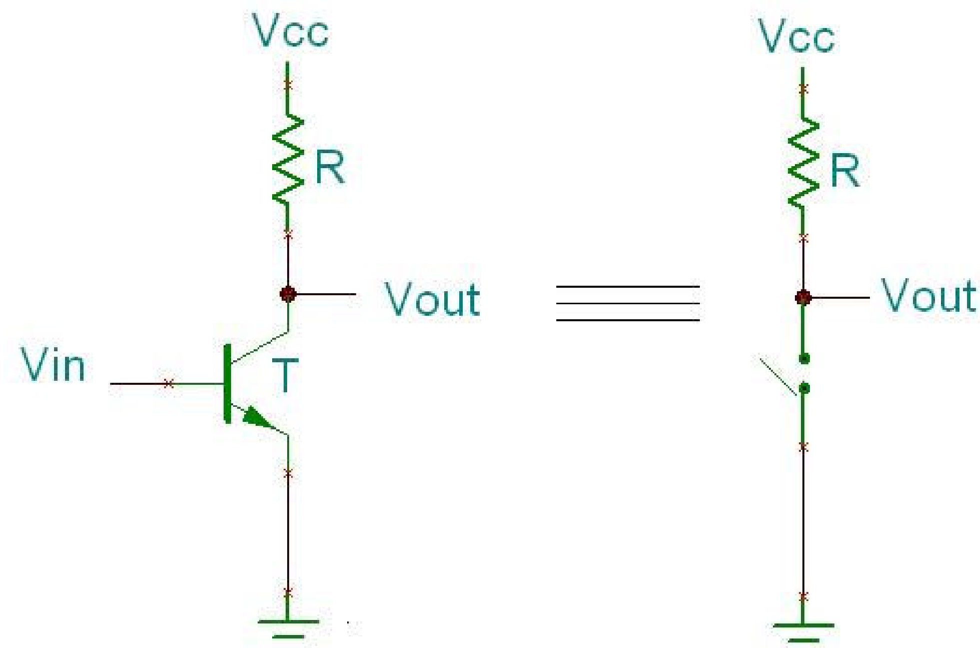
$$V_{BE} = 0.060 \log\left(\frac{I_C}{10^{-13}}\right) \quad \text{at } 27°C$$

Amplifying effect! => small change in base current IB has a large amplifying effect on currents IC and IE

Transistors are active components with the *ability to amplify electrical signal*. Small current at the base B is amplified to produce large current at collector C and emitter E. Transistors are made typically from Silicon (Si) and they come in different categories:

- bipolar (typically analog, range of currents, voltages, frequencies
- field effect (both analog and digital; high impedance
- MOS or CMOS (digital, high speed and low power, respectively)

TRANSISTOR AS A SWITCH



If V_{in} is high, T is ON, switch is closed and V_{out} is low. Digital “0”

If V_{in} is low, T is OFF, switch is open and V_{out} is high. Digital “1”

Switch function occurs when high base voltage (>0.7 V) saturates the transistor and it fully conducts current in the C-E path resulting in Vout =0.

Or when the the base voltage is negative. Then it cuts off the current in the C-E path and Vout =Vcc.

This is the means by which digital or on/off switching can be accomplished and forms the basis for all digital circuits (including computers)

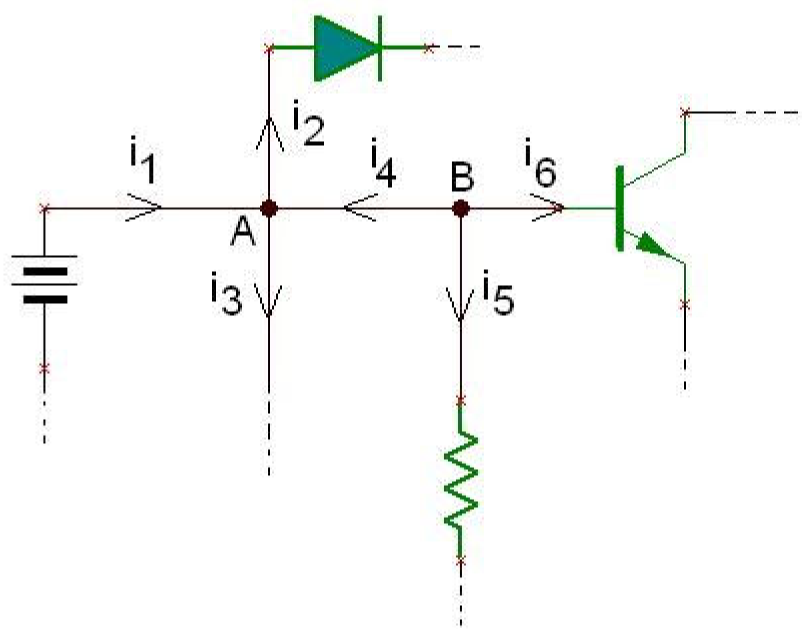
BASIC CIRCUIT LAWS

- Ohm's law – links voltage with current and resistance
  - Know about energy/ power relationships

- Thevenin equivalent circuits
  - Provides means to combine sources of signal (voltage or current, as well as indirectly resistance)

- Kirchoff's current law
  - Provides means to analyze complex circuits
  - Voltages in a loop or current at any node

CIRCUIT ANALYSIS : KIRCHOFF’S CURRENT LAW



Irrespective of the components attached, the sum of all currents entering a node is zero.

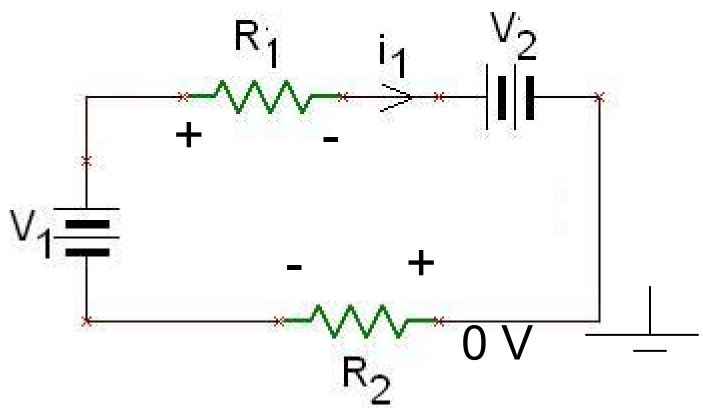
1. Current into node : +ve

2. Current leaving node : -ve

Therefore at A, $\sum I = 0 = i_1 + (-i_2) + (-i_3) + i_4 \Rightarrow i_2 + i_3 = i_1 + i_4$

Therefore at B, $\sum I = 0 = -i_4 - i_5 - i_6 \Rightarrow i_4 + i_5 + i_6 = 0$

CIRCUIT ANALYSIS : KIRCHOFF'S LOOP LAW



In a closed loop, the sum of all voltage drops is zero.

1. Current travels from higher potential to lower.

2. Positive current flows from + to – inside a voltage source.

Thus in the loop,

$-V_1$ due to (2)

$+i_1R_1$ due to (1)

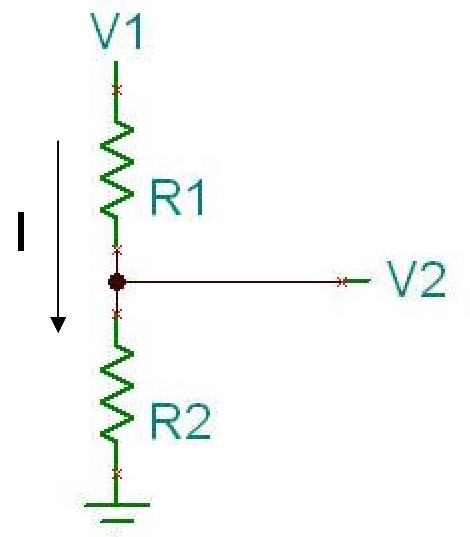
$+V_2$ due to (2)

$+i_1R_2$ due to (1)

$=0$

$$\Rightarrow i_1 = \left(\frac{V_1 - V_2}{R_1 + R_2}\right)$$

VOLTAGE DIVIDER



$$I = \left(\frac{V_1}{R_1 + R_2} \right)$$

$$V_2 = IR_2$$

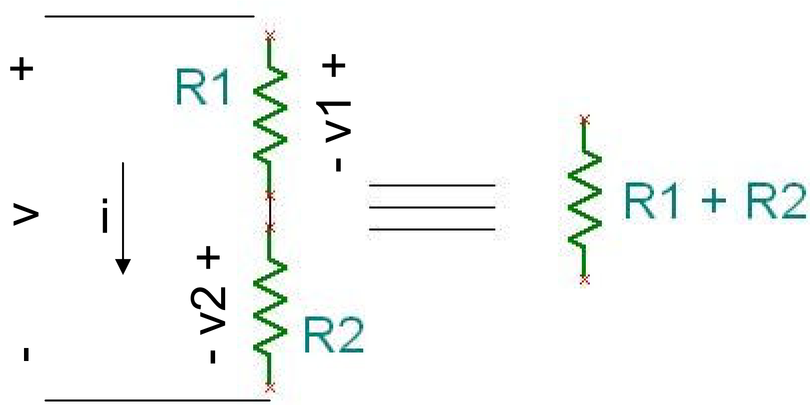
$$\therefore V_2 = V_1 \left(\frac{R_2}{R_1 + R_2} \right)$$

V1 = IR1 + IR2

Therefore, V2 =?

V2 is a fraction of V1 determined by the ratio of the two resistances R1 and R2

COMBINING RESISTORS



Series: from Kirchoff's 2nd law,

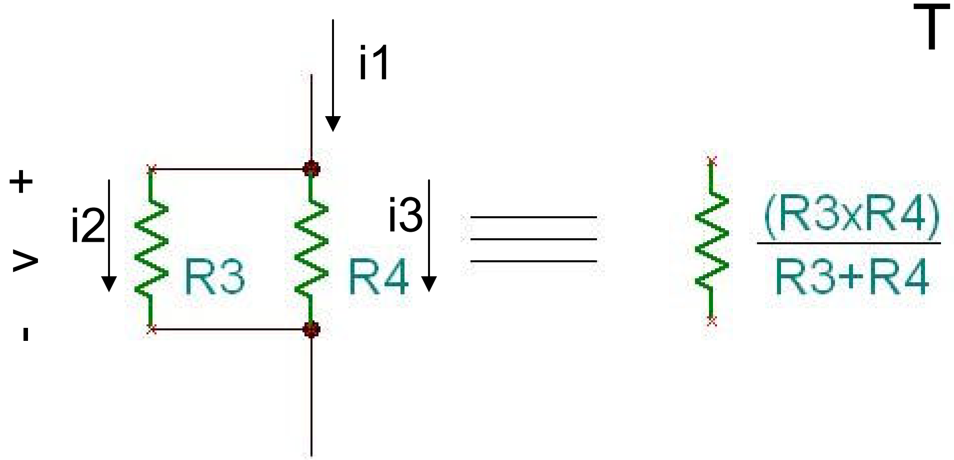
$V - V_1 - V_2 = 0$

$\Rightarrow V = V_1 + V_2$

$\quad = iR_1 + iR_2 = i(R_1 + R_2)$

also $V = iR = i(R_1 + R_2)$.

Thus $R = R_1 + R_2$. [ergo]



Parallel:from Kirchoff's Ist law,

$i_1 = i_2 + i_3$

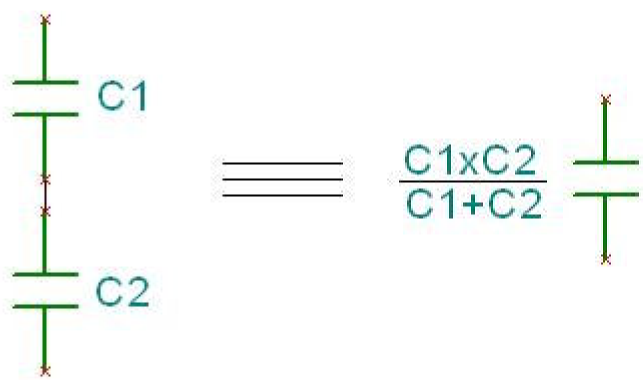
$\quad = (V/R_3) + (V/R_4)$

$\quad = V(R_3R_4/(R_3+R_4))$

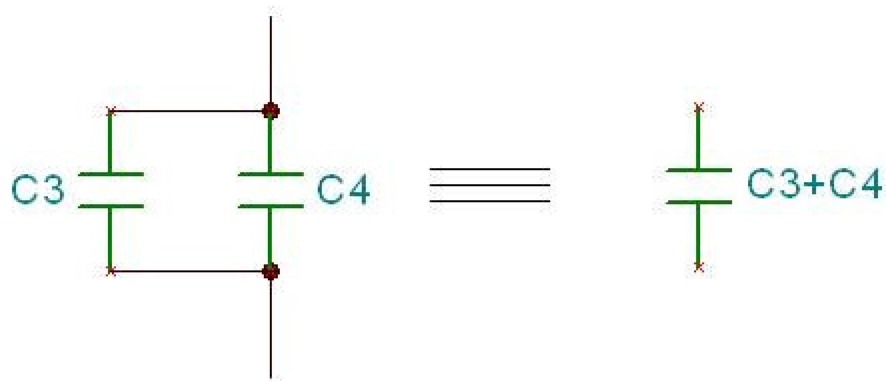
also $i_1 = V/R$

Thus $R = R_3R_4/(R_3+R_4)$. [ergo]

COMBINING CAPACITORS

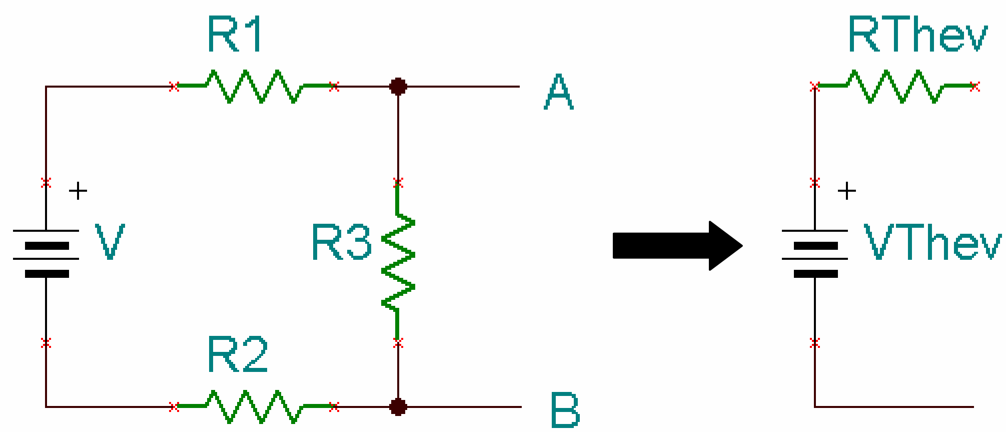


Series



Parallel

THEVENIN EQUIVALENT



Any circuit of resistors and sources can be reduced to a single source and resistor – the Thevenin's equivalent.

V_{Thev} is the voltage seen between A and B

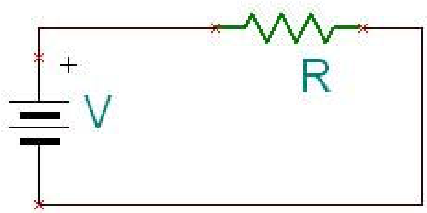
$$= VR_3/(R_1+R_2+R_3)$$

R_{thev} is the resistance seen between A and B with the source shorted

$$= R_3 \,||\, (R_1+R_2) = R_3(R_1+R_2)/(R_3+R_1+R_2)$$

IMPEDANCE

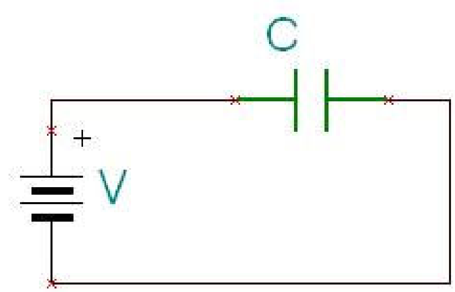
- Different from RESISTANCE as it includes a frequency component – dependent on the frequency f (in Hz) of the input.



$$Z = R$$

Frequency independent

$$Z = \frac{1}{j2\pi fC}$$



Inversely dependent on frequency

RETURN TO CAPACITANCE: FREQUENCY

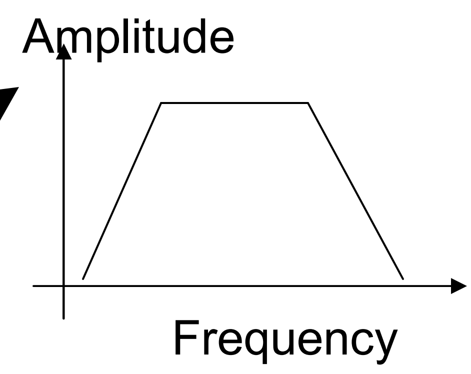
We saw $Z = \frac{1}{j2\pi fC}$ in frequency or fourier domain

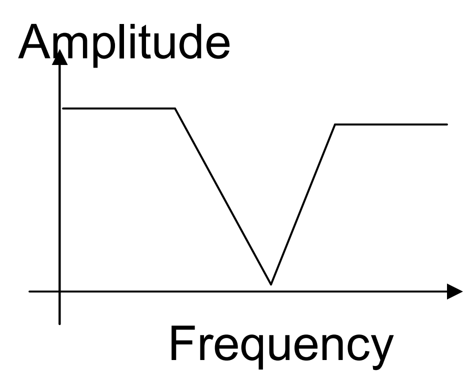
$$Z = \frac{1}{sC}$$ in Laplace domain

Thus for analyzing circuits with capacitors, we assume them to have the above resistance. So the result we get is frequency dependant. This is called the *frequency response* of circuits. We will encounter this in filters.

DIFFERENT FILTERS

- Low pass filter
- High pass filter
- Bandpass filter
  - e.g. ECG: 0.05-100 Hz
- Band reject (notch) filter: e.g. 60 Hz





FILTERS: TRANSFER FUNCTION

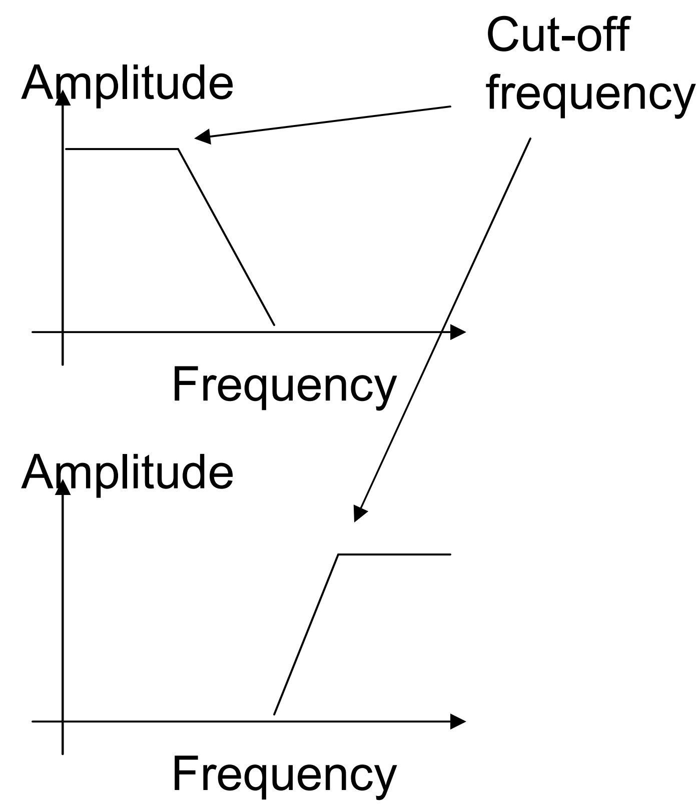
The V_{OUT}/V_{IN} of circuits is called their *Transfer Function*. It is a function of the frequency of operation. It is denoted by H(f). Thus,

For LPF

$$H(f) = \left(\frac{1}{1 + j2\pi fRC} \right)$$

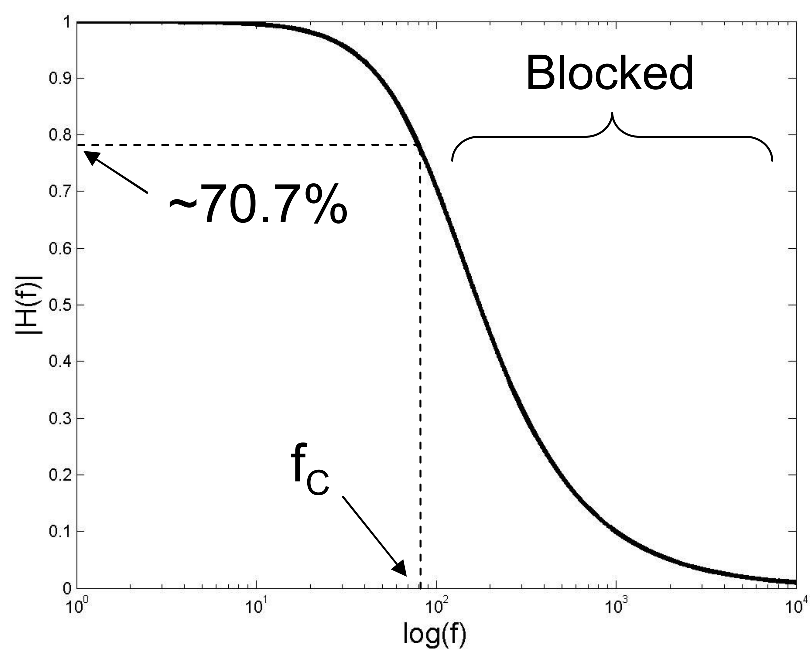
And for HPF

$$H(f) = \left(\frac{j2\pi fRC}{1 + j2\pi fRC} \right)$$



FILTERS: FREQUENCY RESPONSE

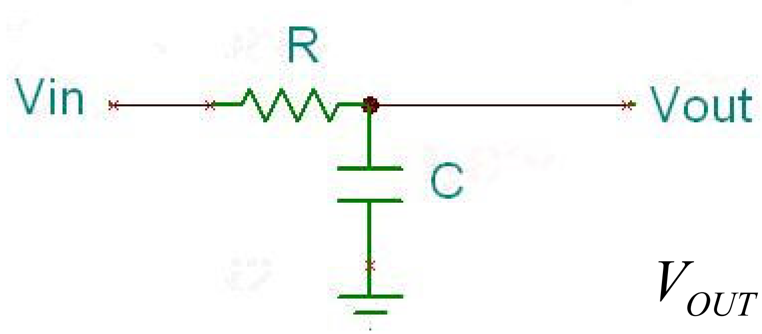
The transfer function exists in the complex number domain. A plot of H(f) is a powerful way to analyse it. There are 2 parts to such a plot (called Bode plot). A magnitude part and a phase part. Shown is a magnitude Bode plot of a LPF.



x axis: log(frequency)
y axis: absolute value of H(f) = |H(f)|

f_C is the frequency on one side of which H(f)~0 or the circuit blocks those frequencies. It is the *cutoff frequency*.

FILTERS: FREQUENCY SENSITIVE CIRCUITS

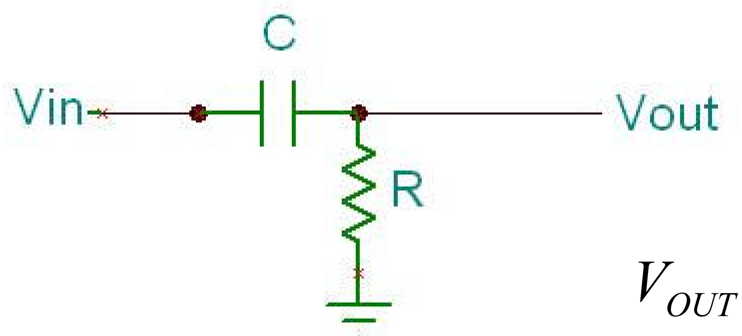


Recall voltage divider,

$$V_{OUT} = V_{IN}\left(\frac{\frac{1}{j2\pi fC}}{R+\frac{1}{j2\pi fC}}\right) = V_{IN}\left(\frac{1}{1+j2\pi fRC}\right)$$

As f increases from 0 onwards, V_{OUT}/V_{IN} (the *TRANSFER FUNTION*) goes from 1 to 0. Thus high frequencies are attenuated. Thus this is a **LOW PASS FILTER.**

FILTERS: FREQUENCY SENSITIVE CIRCUITS



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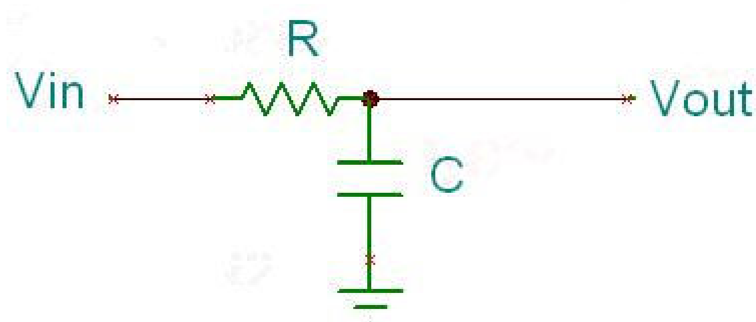
As *f* increases from 0 onwards, V_{OUT}/V_{IN} (the *TRANSFER FUNTION*) goes from 0 to 1. Thus low frequencies are attenuated. Thus this is a **HIGH PASS FILTER.**

FILTERS: FREQUENCY SENSITIVE CIRCUITS – LPF



Recall voltage divider, impedance

$V_{OUT}/V_{IN} = Z_C/(Z_R+Z_C)$

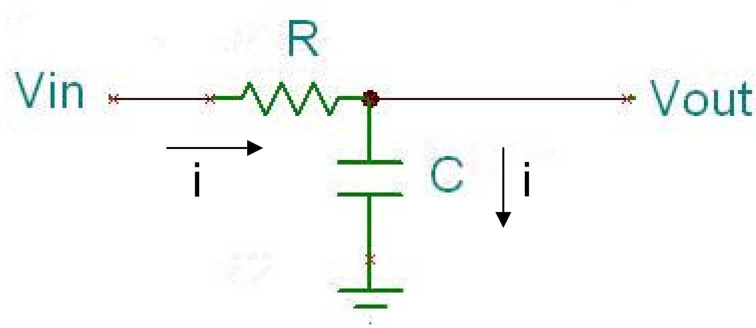
$Z_R = R$

$Z_C = 1/j2\pi f$

$$V_{OUT} = V_{IN}\left(\frac{\frac{1}{j2\pi fC}}{R+\frac{1}{j2\pi fC}}\right) = V_{IN}\left(\frac{1}{1+j2\pi fRC}\right)$$

As f increases from 0 onwards, V_{OUT}/V_{IN} (the *TRANSFER FUNTION*) goes from 1 to 0. Thus high frequencies are attenuated. Thus this is a **LOW PASS FILTER.**

LPF: TIME AND LAPLACE DOMAIN



$(V_{IN} - V_{OUT})/R = i = C dV_{OUT}/dt$

$V_{OUT} + RC dV_{OUT}/dt = V_{IN}$ (1)

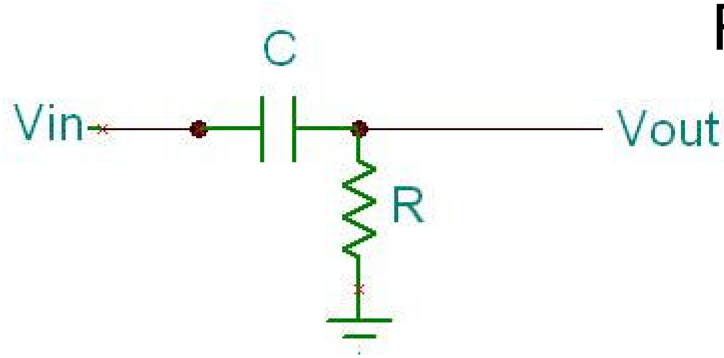
The differential eqn. relating input and output.

Taking Laplace transform of (1),

$V_{OUT}(s) + sRCV_{OUT}(s) = V_{IN}(s)$

$H(s) = V_{OUT}(s)/V_{IN}(s) = 1/(1 + sRC)$ $[= H(f)$ for $f = j\omega = j2\pi f]$

FILTERS : FREQUENCY SENSITIVE CIRCUITS – HPF



Recall voltage divider, impedance

$V_{OUT}/V_{IN} = Z_R/(Z_R+Z_C)$

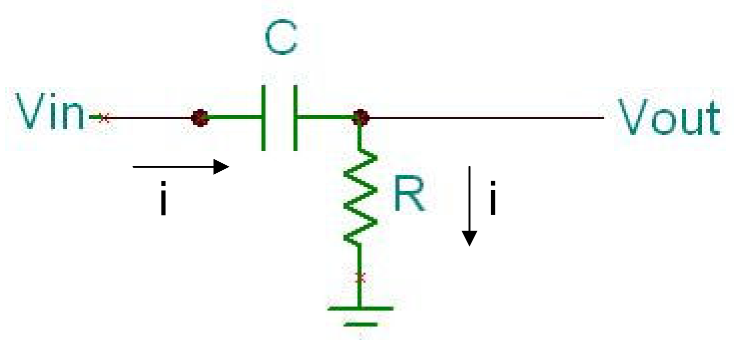
$Z_R = R$

$Z_C = 1/j2\pi f$

$$V_{OUT} = V_{IN}\left(\frac{R}{R+\frac{1}{j2\pi fC}}\right) = V_{IN}\left(\frac{j2\pi fRC}{1+j2\pi fRC}\right)$$

As *f* increases from 0 onwards, V_{OUT}/V_{IN} (the *TRANSFER FUNTION*) goes from 0 to 1. Thus low frequencies are attenuated. Thus this is a **HIGH PASS FILTER.**

HPF: TIME AND LAPLACE DOMAIN



$i = C\,d(V_{IN} - V_{OUT})/dt = V_{OUT}/R$

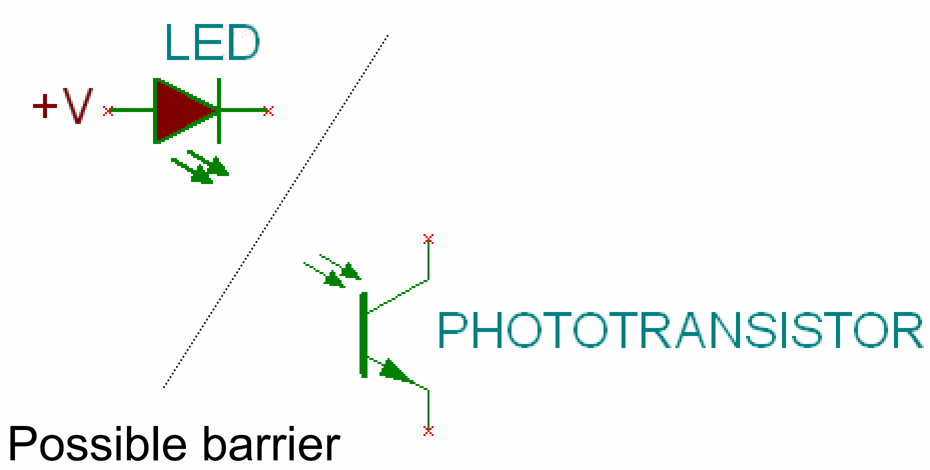
$V_{OUT} + RC\,dV_{OUT}/dt = RCV_{OUT}/dt$

Taking Laplace transform,

$V_{OUT}(s) + sRCV_{OUT}(s) = sRCV_{IN}(s)$

$H(s) = V_{OUT}(s)/V_{IN}(s) = sRC/(1 + sRC)$ [= H(f) for f = jω = j2πf]

OPTOELECTRONICS



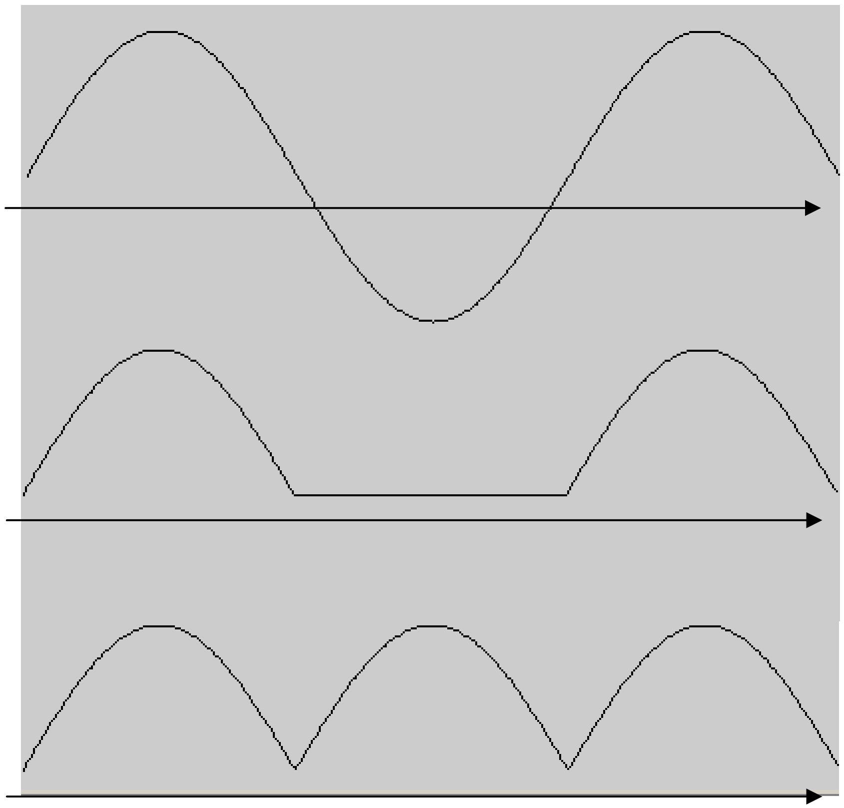
A medical application: shining an LED into finger and using a phototransistor to detect reflected or transmitted light to pick up finger pulse.

LED : light emitting diode

Phototransistor : light sensitive device made from Si

Recall Transistor as a switch. If light falls on the transistor, it is a closed switch else open. Can use to detect barriers – burglar alarms etc.

RECTIFIERS



Original input

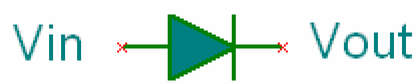
Half wave rectified output

Full wave rectified output

Rectification is done using diodes. Rectifiers are useful in power supplies: to convert ac signals to dc. Ac to dc conversion is done by rectifying and then peak voltage stored on a capacitor to produce dc voltage from ac.

RECTIFIERS

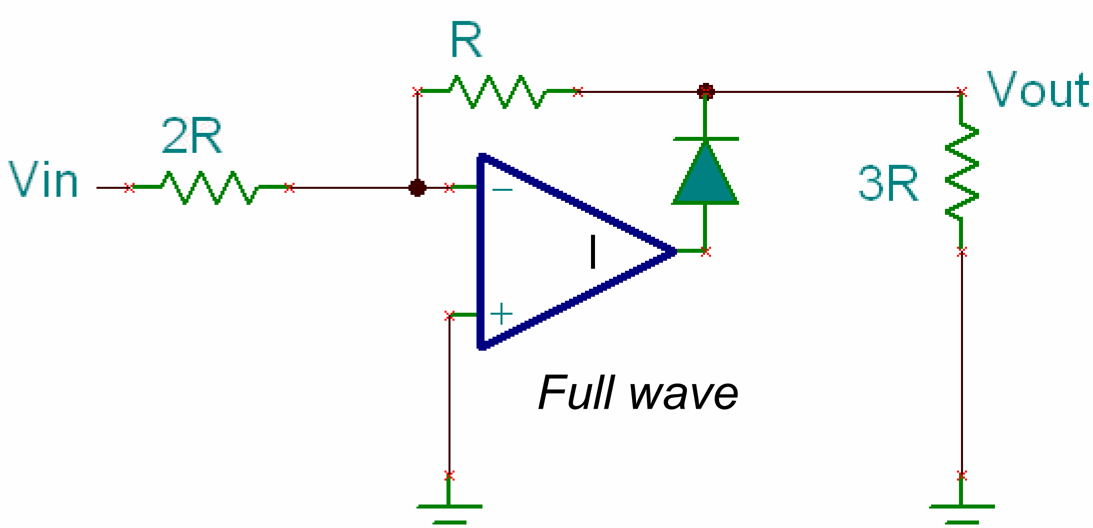
Recall Diodes, inverting amplifier, comparator, voltage divider.



If $V_{IN}>0$, $V_{OUT} = V_{IN}$,

If $V_{IN}<0$, $V_{OUT} = 0$

Half wave

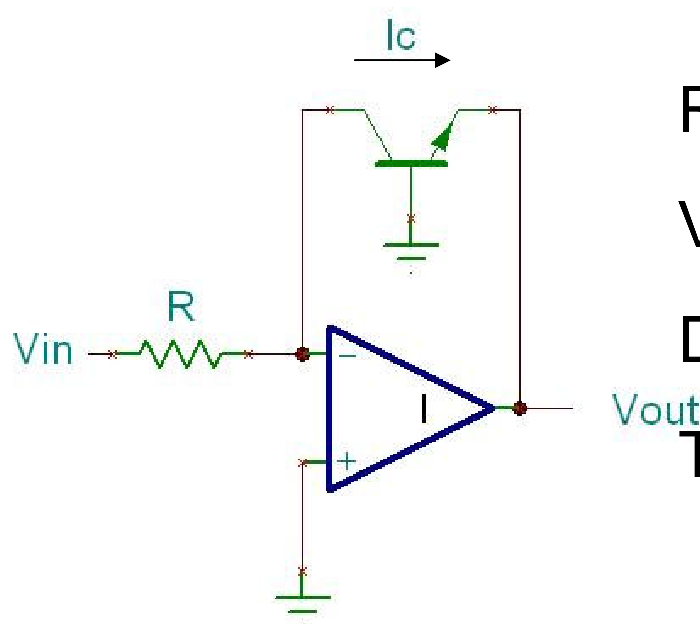


Full wave

If $V_{IN}>0$, $A = -V_{ee}$, diode is off, $V_{OUT} = V_{IN}(3R/(2R+R+3R)) = V_{IN}/2$ (divider)

If $V_{IN}<0$, $A = V_{cc}$, diode is on, $V_{OUT} = -V_{IN}(R/2R) = -V_{IN}/2$ (inverting amplifier)

NONLINEAR AMPLIFIER



Recall BJT. $V_{OUT} = -V_{BE}$

$V_{BE}=0.06\log(I_C/10^{-13})$

Due to opamp, $I_C = V_{IN}/R$

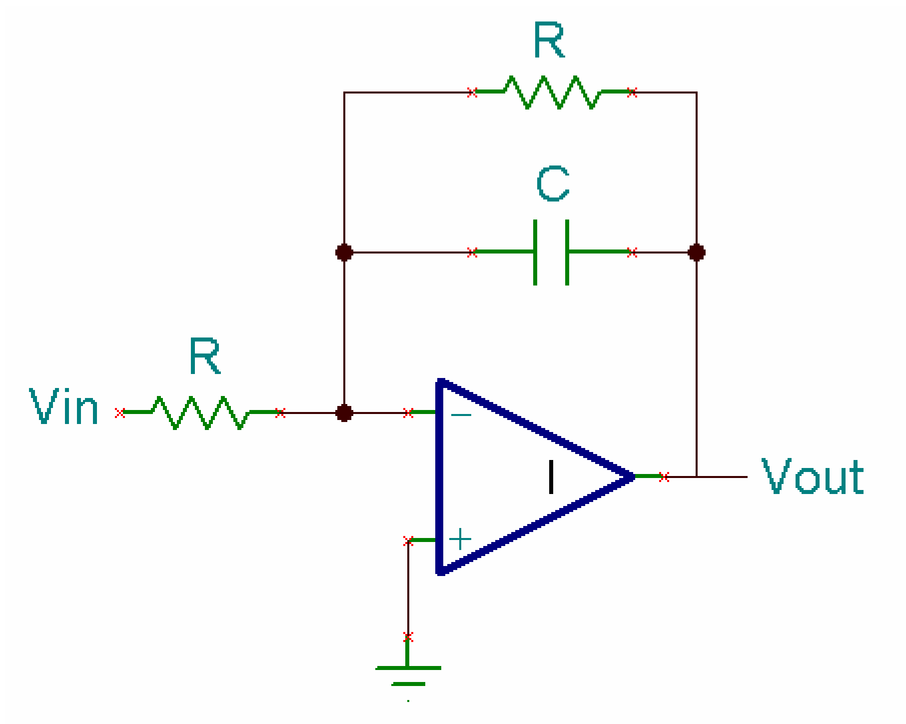
Thus $V_{OUT} = -0.06\log(V_{IN}/10^{-13}R)$

What are the possible applications of nonlinear circuits?

e.g. temperature dependent transistor, doing logarthmic calculations

eg. linearizing other nonlinear circuit components

ACTIVE FILTERS



Recall impedance, inverting amplifier.

The feedback resistance is R in parallel with $1/j2\pi fC$

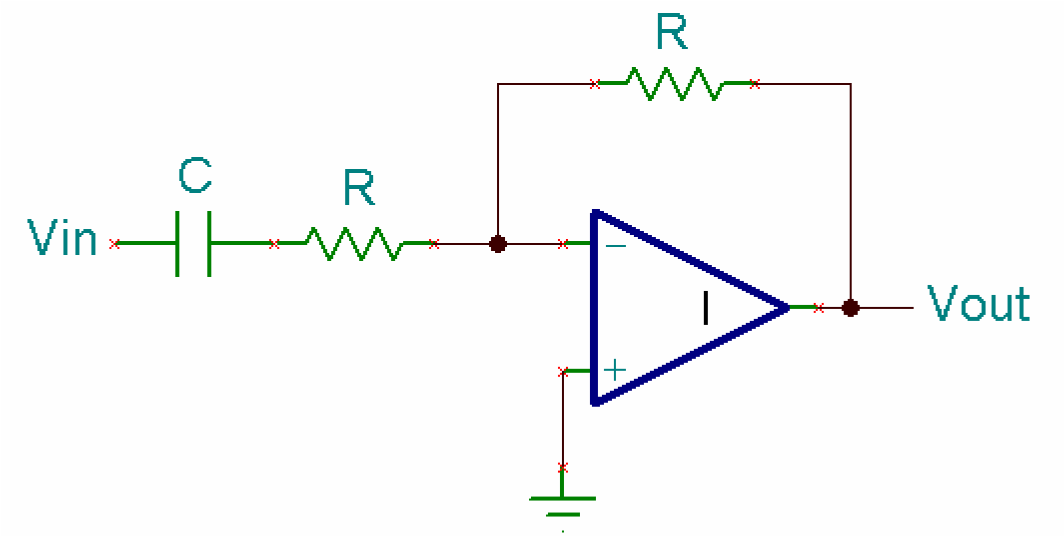
$= R/(1+j2\pi fRC)$

$V_{OUT}/V_{IN} = R_2/R_1 = R/(1+j2\pi fRC)R = 1/(1+j2\pi fRC)$

$= 1$ at $f = 0$ and $= 0$ as $f \to \infty$ therefore LPF

Cutoff frequency $f = 1/\ 2\pi\ \mathit{sqrt}(RC)$ (where $|H(f)|=0.707$)

ACTIVE FILTERS



Resistance R is in parallel with the capacitor, impedance is R in series with $1/j2\pi fC$

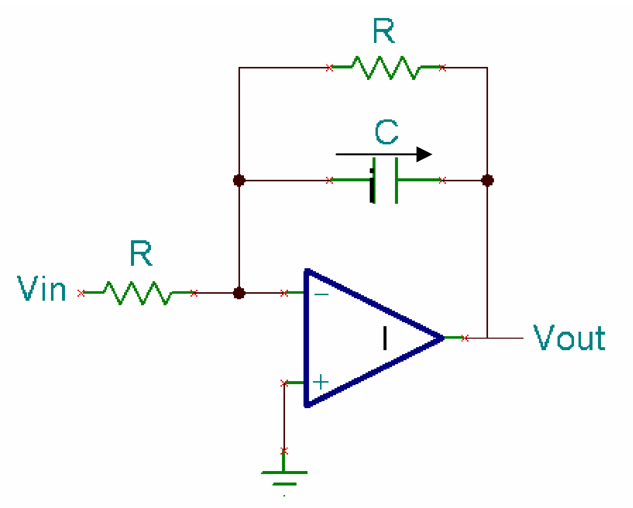
$= R+1/ j2\pi fC$

$V_{OUT}/V_{IN} = R_2/R_1 = j2\pi fRC/(1+j2\pi fRC)$

$= 0$ at $f = 0$ and $= 1$ as $f \rightarrow \infty$ therefore HPF

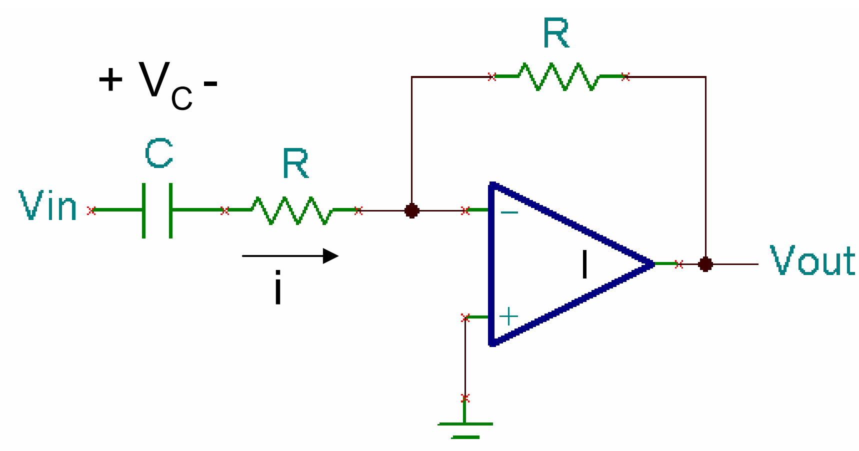
Cutoff frequency $f = 1/ 2\pi\ \mathit{sqrt}(RC)$ (where $|H(f)|=0.707$)

FILTERS AS INTEGRATORS AND DIFFERENTIATORS



LPF, $V_{OUT} = 0 - \frac{1}{C}\int i\,dt$

Thus it is like an integrator



HPF, $V_{OUT} = 0 – iR = -RC\ dV_C/dt$

Thus it is like a differentiator

Electric Circuits - Overview

- What are the different electrical circuit components?
  - R, C, diode, transistor, transformer, inductor…
- What are the circuit analysis laws?
  - What is Ohm’s law
  - Kirchoff’s current law (current at a node sums to zero): think of water pipes distributing water through finer pipes at a node
  - Kirchoff’s voltage law (voltage around a loop sums to zero): think of going up and down the hill and gaining/loosing potential energy
- Develop simple circuits (buy parts at an electronic store such as XYZ Shack and build hobby prototypes)
  - Battery voltage applied to a beeper (match voltages)
  - Battery voltage applied to an electric motor (“) via a switch
  - Battery connected via a resistor to an LED
  - Develop time-varying circuits using capacitors
  - Calculate the time constant, time varying charge, discharge equation

Electric Circuits - Overview

- Semiconductors such as Si and Ge

- Diode: current in one direction
  - Come up with one or two applications of diodes

Transistor: amplifies current (current in the collector-emitter path is amplified version of the current in the base)

- Use of a transistor to amplify audio sound: interface a microphone and a loud speaker

Operational amplifier

- Ideal op amp, Real op amp
- Use an op amp to compare a signal with a threshold voltage
- Use an op amp to amplify low level microphone signal

Case Studies

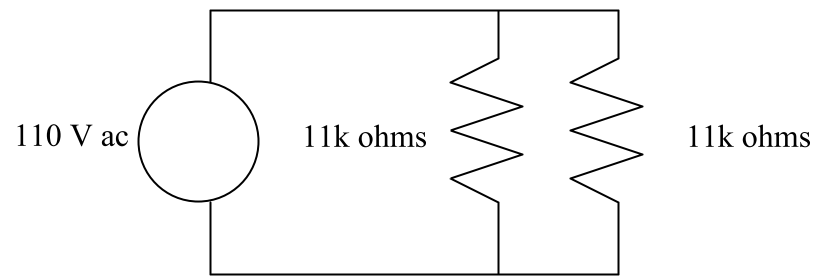
Applications of dc circuits

- Voltage across a cell membrane
- Ion channel currents
- Voltage clamp
- Patch clamp

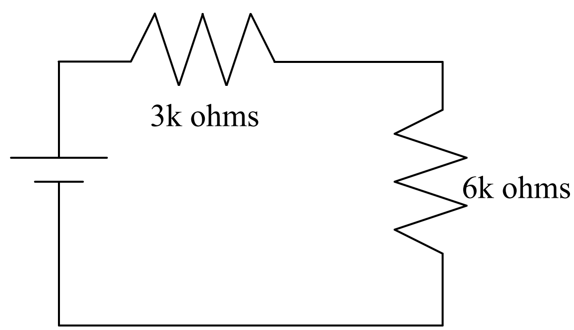
Applications of time varying circuits

- Charging and discharging a capacitor
- Charging and discharging a defibrillator

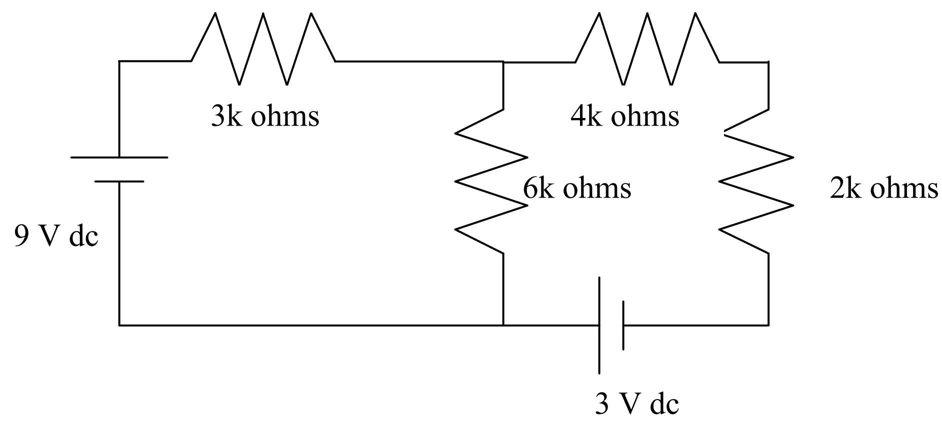
Calculate the current in the following circuit. What is the equivalent resistance of the two resistors in parallel?



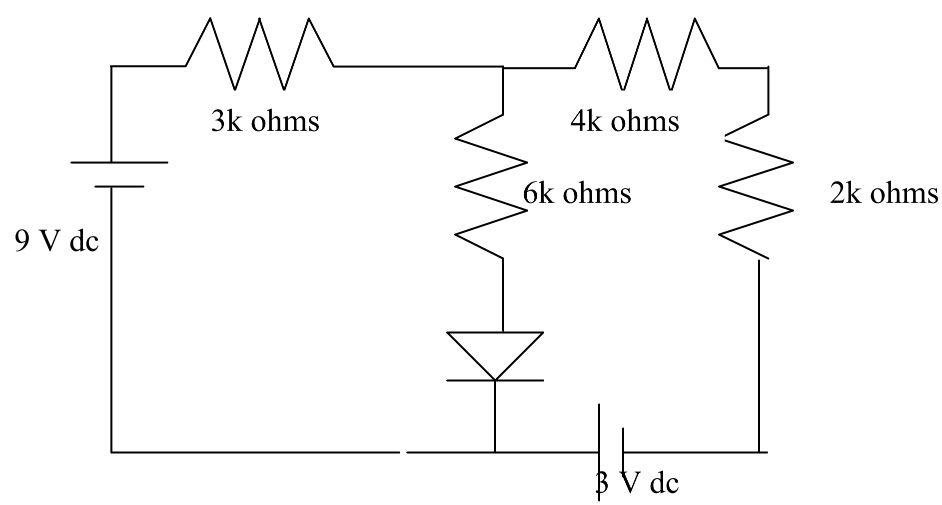
Calculate the voltage across each resistor. What is the equivalent resistance value of the two resistors in series?

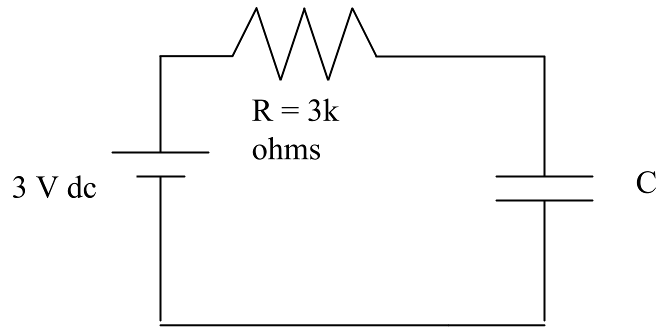


Use Kirchoff's current law and voltage law to calculate the current through and voltage across each of the resistors.



In the following circuit, put a switch so that the LED would light up when turned on. Will this LED light up (hint: find the current direction)?





Calculate the rate of charging (time constant) for the following circuit. What should be the value of the capacitor to achieve 10 second time constant? How long will it take for the capacitor to "fully" charge? What will be the current in the circuit when the capacitor is fully charged?

- Design a defibrillator circuit as follows. We want to deliver a 1000 Volt pulse for 10 ms when the physician commands it. This would be done in two steps. The first step is the "charge" command, and the second step is the discharge or shock command.

- Draw the shape of the defibrillator pulse that was produced by a charging a capacitor of 100 mF through a resistor of 100 ohms and then discharged through a resistance of 1 k ohm.

- In the class, we gave the formula for power as P = VI and the energy as E = VI DT. Derive the formula for energy in terms of voltage and resistance R. That is, the current I is flowing through resistance R when voltage V is applied.

- Calculate the energy consumed by a pacemaker which has to produce a 1 ms pulse of 5 Volt amplitude for 10 years. The pacemaker delivers the pulse in a heart with 50 ohm resistance.

Draw the circuit of a differentiator –OR– integrator.

- Now, through circuit analysis show why that circuit works like a differentiator/ integrator (i.e. derive the relationship between output and input of the amplifier to show using your equation that the input signal is differentiated/integrated.

- Next, obtain the frequency response of this circuit. E.g. you can derive a transfer function (output over input) in the Laplace form, substitute s=jw and then show the frequency response.

- Point out *one major advantage* and *one major disadvantage* of an analog differentiator/integrator over a digital/software version.

- Describe (*very briefly*) a biomedical instrumentation application of an integrator or differentiator.